Topic: Mean value theorem

Question: Which is the correct statement of the Mean Value Theorem?

Answer choices:

A If f is continuous and differentiable on the open interval (a, b), then there exists a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

B If f is continuous on the closed interval $[a, b]$, then there exists a number c in $[a, b]$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

C If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists a number c in $[a, b]$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

D If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

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Solution: D

The correct statement of the mean value theorem is:

> If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists a number c in (a, b) such that
>
> $$f'(c) = \frac{f(b) - f(a)}{b - a}$$

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Topic: Mean value theorem

Question: True or false?

Two police officers are sitting separately along a highway, 3 miles apart. A car passing the first officer is clocked going 60 mph. Two minutes later, the car is clocked going 58 mph by the second officer. Based on this information, they can prove that the car was speeding (going over the 65 mph speed limit) at some point in between them.

Answer choices:

A True

B False

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Solution: True

The officers can prove that the car was traveling faster than the 65 mph speed limit by using the mean value theorem. There is some point along the 3-mile stretch where the car must have been traveling at an instantaneous speed equal to the average speed over the 3-mile stretch.

Let time $t = 0$ and position $s = 0$ when the car passes the first officer, and let $t = 2/60 = 1/30$ hr and $s(1/30) = 3$ miles. Then the average speed of the car over the 3 miles and 2 minutes is

$$v_{avg} = \frac{s\left(\frac{1}{30}\right) - s(0)}{\frac{1}{30} - 0}$$

$$v_{avg} = \frac{3 - 0}{\frac{1}{30}}$$

$$v_{avg} = 3 \cdot \frac{30}{1}$$

$$v_{avg} = 90 \text{ mph}$$

Therefore, the car was traveling at 90 mph at some point along the 3-mile stretch.

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Topic: Mean value theorem

Question: Which one of the following statements is true?

Answer choices:

A The Mean Value Theorem for Derivatives applies to functions that are continuous and differentiable on a given interval $[a, b]$ only and states that there will be a point c in the interval such that $f(b) - f(a) = f'(c)(a - b)$.

B The Mean Value Theorem for Derivatives applies to functions that are discontinuous and differentiable on a given interval $[a, b]$ only and states that there will be a point c in the interval such that $f(b) - f(a) = f'(c)(b - a)$.

C The Mean Value Theorem for Derivatives applies to functions that are continuous and differentiable on a given interval $[a, b]$ only and states that there will be a point c in the interval such that $f(b) - f(a) = f'(c)(b - a)$.

D The Mean Value Theorem for Derivatives applies to functions that are discontinuous and differentiable on a given interval $[a, b]$ only and states that there will be a point c in the interval such that $f(b) - f(a) = f'(c)(a - b)$.

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Solution: C

The Mean Value Theorem for derivatives applies to functions that are continuous and differentiable on a given interval $[a, b]$ only and states that there will be a point c in the interval such that

$$f(b) - f(a) = f'(c)(b - a)$$

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